NAG Toolbox for MATLAB

f07ue

1 Purpose

f07ue solves a real triangular system of linear equations with multiple right-hand sides, $AX = B$ or $A^{T}X = B$, using packed storage.

2 Syntax

```
[b, info] = f07ue(uplo, trans, diag, ap, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07ue solves a real triangular system of linear equations $AX = B$ or $A^{T}X = B$, using packed storage.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J 1989 The accuracy of solutions to triangular systems *SIAM J. Numer. Anal.* **26** 1252–1265

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates whether A is upper or lower triangular.

uplo = 'U'

A is upper triangular.

uplo = 'L'

A is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **trans – string**

Indicates the form of the equations.

trans = 'N'

The equations are of the form $AX = B$.

trans = 'T' or 'C'

The equations are of the form $A^{T}X = B$.

Constraint: **trans** = 'N', 'T' or 'C'.

3: **diag – string**

Indicates whether A is a nonunit or unit triangular matrix.

diag = 'N'

A is a nonunit triangular matrix.

diag = 'U'

A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = 'N' or 'U'.

4: **ap**($*$) – **double array**

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$.

The n by n triangular matrix A, packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in $\mathbf{ap}(i + j(j-1)/2)$ for $i \leq j$;
if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in $\mathbf{ap}(i + (2n-j)(j-1)/2)$ for $i \geq j$.

If **diag** = 'U', the diagonal elements of A are assumed to be 1, and are not referenced; the same storage scheme is used whether **diag** = 'N' or 'U'.

5: **b**(**ldb**,$*$) – **double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n** – **int32 scalar**

Default: The first dimension of the array **ap** and the second dimension of the array **ap**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p** – **int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldb

5.4 Output Parameters

1: **b**(**ldb**,$*$) – **double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r solution matrix X.

2: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **trans**, 3: **diag**, 4: **n**, 5: **nrhs_p**, 6: **ap**, 7: **b**, 8: **ldb**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, $a(i,i)$ is exactly zero; A is singular and the solution has not been computed.

7 Accuracy

The solutions of triangular systems of equations are usually computed to high accuracy. See Higham 1989.

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A+E)x=b$, where

$$|E| \leq c(n)\epsilon|A|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A,x)\epsilon, \qquad \text{provided} \qquad c(n)\,\mathrm{cond}(A,x)\epsilon < 1,$$

where $\mathrm{cond}(A,x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty$.

Note that $\mathrm{cond}(A,x) \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$; $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$ and it is also possible for $\mathrm{cond}\left(A^{\mathrm{T}}\right)$ to be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling f07uh, and an estimate for $\kappa_\infty(A)$ can be obtained by calling f07ug with **norm_p** $=$ 'I'.

8 Further Comments

The total number of floating-point operations is approximately n^2r.

The complex analogue of this function is f07us.

9 Example

```
uplo = 'L';
trans = 'N';
diag = 'N';
ap = [4.3;
     -3.96;
     0.4;
     -0.27;
     -4.87;
```

```
     0.31;
     0.07000000000000001;
     -8.02;
     -5.95;
     0.12];
b = [-12.9, -21.5;
     16.75, 14.93;
     -17.55, 6.33;
     -11.04, 8.09];
[bOut, info] = f07ue(uplo, trans, diag, ap, b)
```

```
bOut =
   -3.0000   -5.0000
   -1.0000    1.0000
    2.0000   -1.0000
    1.0000    6.0000
info =
           0
```